THE LION CAPITAL OF UDAYAGIRI AND ITS PILLAR DEPICT A VERY HIGH STATE OF ASTRONOMICAL KNOWLEDGE IN ANCIENT INDIA

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ABSTRACT

In this work the famous Lion Capital of Udayagiri and its Pillar are closely analyzed. The pillar has a lion on the top with a circular base on which animals / birds are shown. This pillar is highly decorated and has octagonal and 16- sided polygon portions.

The work shows that each and every part of this pillar depicts high state of knowledge of mathematics / astronomy in ancient India. This work proves beyond doubt that Udayagiri was used for ancient astronomical observations.

KEYWORDS: Ancient astronomy in India, Udayagiri Lion Capital, Prahars as a measure of time, Octagonal and 16-sided pillar

NOMENCLATURE

α	Angle in vertical plane
α_s	Angle in horizontal plane
γ	Latitude of the place
δ	Declination
h_s	Hour angle at any instant of time
h_{sr}	Hour angle at sunrise measured from noon
N	Day number of a year

1. INTRODUCTION

Many researchers such as Dass, Willis, Balasubramanium [1-5] have published several papers about Udayagiri near Sanchi in Madhya Pradesh, India as being an astronomical site. One of such papers was written by Dass and Willis [7]. This deals with the Lion Capital of Udayagiri where they discuss that the pillar has a lion on the top and 16-sided middle portion whereas its bottom part is of an octagonal shape. This portion was seen by the author of this paper lying at the end of the passageway (see its location at Number 3 in Fig 2A here). In [7] the octagon is shown as Fig. 10. It is about 1 meter long, and a face width is about 15 centimeters.

The lion is mounted on a circular base on which several animals / birds are depicted (see Figs. 1A and 1B here) [6]. Fig. 1B clearly shows that their locations are at the vertices of an inscribed octagon. Thus, these animals have some sort of relationship with the lower octagon.

The octagonal shape is very uncommon in India. A sketch of this pillar is shown in Fig. 3 where the pillar is mounted on an elevated platform. The lion faces due east unlike the Vishnu's statue lying in the Kshirsagar in cave 13. In this case, it is inclined at an angle in the horizontal plane - equal to that of the rising sun on the summer solstice day [8].

The objective of this work is to explain the unusual octagonal positions of the animals and their relationship with the lower portion of the octagonal part of the pillar. In other words, the objective is to explain what do these animals / birds represent?

It is also an objective of this work to show the importance of 16-sided polygon on a portion of the pillar, and the concept of scientific accuracy amongst the scientists / astronomers of ancient India.

One can see in Fig. 2 that the pillar's location was at Number 1 and it is on a flat area where one could make astronomical observations given the fact that Udayagiri lies on the Tropic of Cancer where the sun shines over the head on the summer solstice day. One can also see a platform at some distance in the east shown as Number 2 in Figs 2A, 2B, and 12.

2. THEORETICAL CONSIDERATIONS

If we refer to Fig. 4, one can calculate δ (the declination), αs (angle from south in the horizontal plane) and α (the angle in the vertical plane) using the following formulas [9-11]:

$$\delta = 23.45 \sin \{ (360 / 365) (284 + N) \} \quad (1)$$

Where δ is the declination of the sun in degrees, and N is the day number, which is the number of that day in a year. For example, on January 1, N is equal to 1.

The time of sunrise, h_{sr} in hour-angle from the noon, is calculated from

$$h_{sr} = \cos^{-1} \{ - \tan(\delta) \tan(\gamma) \} \quad (2)$$

where, γ is the latitude. Denoting the instant of time in terms of the hour angle from the noon as hs , one can write

$$\sin(\alpha) = \cos(\gamma) \cos(\delta) \cos(hs) + \sin(\gamma) \sin(\delta) \quad (3)$$

and by expressing the angles in degrees, one can write

$$\alpha_s = \sin^{-1} \{ \cos(\delta) \sin(hs) / \cos(\alpha) \} \quad (4)$$

if

$\cos(hs) > \{\tan(\delta) / \tan(\gamma)\}$

or

$\alpha_s = 180^o - \sin^{-1}\{\cos(\delta)\sin(hs) / \cos(\alpha)\}$ (5)

if

$\cos(hs) < \{\tan(\delta) / \tan(\gamma)\}$

In this paper, we are interested in calculating h_{sr} (the time of sunrise), h_s and α_s, the angle from the south. This is because it is the horizontal component of the sunray that results in the illumination of a vertical surface such as the octagonal pillar; the vertical component is parallel and would not result in the illumination of such a surface.

2.1 VIEW OF THE PILLAR ON THE SUMMER SOLSTICE DAY

The choice of a shape of a pillar is to get a vertical side (since the rays at sunrise are horizontal) parallel to the sun's rays, as close as possible. Secondly, the position of the rising sun on any day depends upon the inclination of the earth's spin axis with respect to the ecliptic. This applies to the solstices and equinoxes also.

Fig. 5 shows the view at sunrise on this day. Vertices 4 to 1 and 8 will be illuminated as the sun moves in the clockwise direction towards south as shown by the arrows. The sides which will be dark (in a shadow) would be D, E, F, and G. As time passes, side H will become dark and D will get illuminated. None of the sides are even closely parallel to the sun's rays at the time of sunrise in this octagon on this day.

Better accuracy is obtained if one uses 16- sided polygon as shown in Fig. 9. There is about 3 degrees of difference between the orientation of the sun's rays and the side 6-7 on the left hand side which is illuminated. This accuracy is remarkable for its time, and it is an admirable fact that the ancient astronomers could think of achieving such an accuracy.

The sun sets in the north-west as shown in Fig. 5 but it travels through the south at noon.

2.2 VIEW OF THE OCTAGONAL PILLAR ON THE EQUINOX DAY

The results for this day are shown in Fig. 6 which is an interesting figure. Here, the sun rises in the east when sides A, B, and C will be illuminated and the rest will be in dark or in a shadow. At the end of Prahar I (a Prahar in Sanskrit word and is equal to 3 hours), the situation in terms of illumination – will be identical to that of the sunrise i.e. now, sides A and E will be in dark. This process repeats itself every 45 degree rotation of the sun. Therefore, a Prahar is an interval of time when the sun’s rays attain a position with respect to the octagon – identical to the one at sunrise.

Therefore, the subdivision of a day (24 hours) into Prahars (also called Pahars in India, Pakistan, and Nepal) came from this kind of experiment.

On this particular day, the day and night are of equal duration. Thus, the 24 hours between sunrise to sunrise have 8 equal Prahars. On other days, the Prahars of day and night (4 each) will be of unequal lengths of time.

Clearly, we can say that the eight animals / birds shown in the circular part below the lion – represent the eight Prahars of the equinox day because they are spaced at equal angular intervals at the vertices of an octagon.
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2.3 VIEW OF THE PILLAR ON THE WINTER SOLSTICE DAY

Fig. 7 shows that the sunrise takes place after the rays fall on the vertex 1. So, the sides A to D will be illuminated at the sunrise. Since the angular span of the day is much smaller than that of night, it will result into smaller angular spans for the Prahars of day than those at equinox.

However, just like in Fig. 5, one gets better results if 16- sided polygon is used as shown in Fig 8. Here, the difference of angle between the sun's rays and side 12 -13 is about 2 degrees which is fairly accurate.

2.4 THE CHOICE OF POLYGONAL SHAPES

Each side of an octagon subtends an angle equal to 45 degrees at the centre. Therefore, the half of the side will make an angle equal to 22.5 degrees. The inclination of the earth's spin axis to the ecliptic plane is approximately equal to 23.5 degrees just slightly greater than the angle made by half of the side. Therefore, angles made by the sun's rays at the centre of the octagon will almost match with the vertices as arranged in the pillar on each of the solstices whether winter or summer. This concept becomes very useful when using a vertical gnomon which is discussed in the next section.

The 16 - sided polygons offer a sharp change in illumination of a side with respect to time due to blocking off rays - if the angle is within a few degrees of a side. This is because the rotational (spin) speed is about 15 degrees per hour.

On the equinox day, an octagon alone is sufficient as it has two sides parallel to the sun's rays at the time of sunrise.

2.5 USE OF OCTAGONAL SHAPE USING A VERTCAL GNOMON

If one uses a vertical gnomon then the visualization of earth's motion around the sun becomes crystal clear. The idea must have come to someone in Ancient India with a very clear picture of the exact path of motions of earth around the sun. Such an idea was not there in Europe even after Copernicus's time in the 16^{th} century. The path of the earth around the sun at that time in Europe was of a complicated geometry. In the

Copernican heliocentrism , the astronomical model developed by Nicolaus Copernicus and published in 1543, the Sun was positioned near the center of the Universe, motionless, where the earth and the other planets rotated around it in circular paths modified by epicycles and at uniform speeds. Such a model will not yield results displayed through experimentation at Udayagiri with these octagonal and 16- sided polygons. This choice of an octagon or 16-sided polygon in the Udayagiri Lion Pillar makes this fact abundantly clear.

Fig. 10 shows the concept of the use of vertical gnomon with an octagon. In this figure, a vertical gnomon placed at the centre, will cast a shadow near a vertex of an octagon on the summer solstice day since the sun on this day rises close to a vertex on the opposite side of the gnomon. Such gnomons have been in use in Ancient India, and Fig. 11 shows one such use at Ujjain, a center of astronomy since the ancient times.

Fig. 12 shows the platform for observations at Udayagiri (also shown in Figs 2A and 2B) similar to the one in Ujjain.

3 CONCLUSIONS

Based on the work here, the following conclusions can be drawn:

1. The animals/birds depicted below the lion on the Udayagiri Pillar represent Prahars of the equinox day since they are spaced at equal angular intervals.

2. The ancient astronomers of India had a very clear picture of the path of motion of earth around the sun as they chose octagonal shape and a 16-sided polygon portions of the pillar which are symmetrical in shape. The path of the earth around the sun is symmetrical and almost circular.

3. The ancient astronomers were very conscious of scientific accuracy since the sides were found to be parallel to the rising sun on the solstices or equinoxes within 3- degree accuracy.

4 The use of such a pillar (octagon or 16 sided polygon) at Udayagiri removes any doubts that it was a place of ancient astronomical observations.

5 The definition of Prahars or the divisions of a day into eight - came from the use of octagons which encompass the earth's motion around the sun i.e., sun's position on equinoxes. Such a selection of a geometrical shape shows the brilliance of ancient astronomers / mathematicians of India unparallel in this world for its time.

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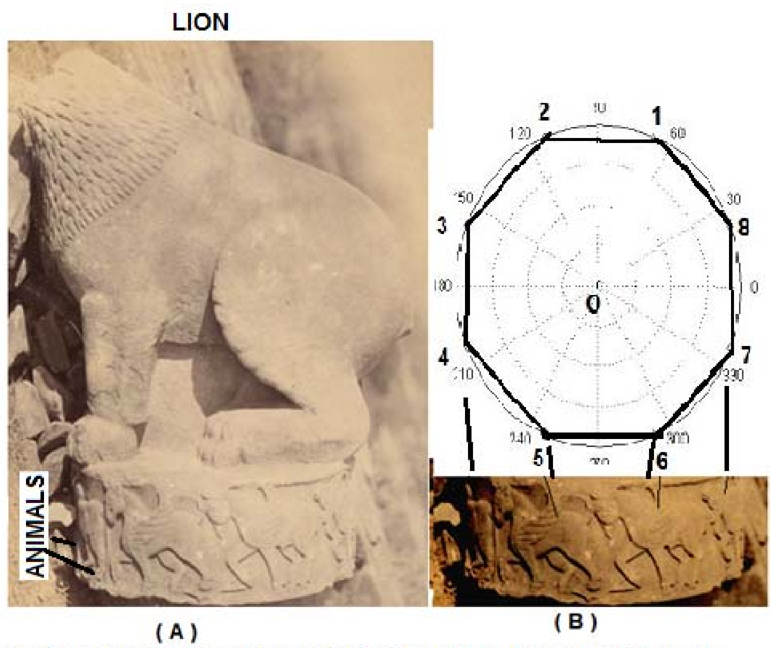

FIG. 1 UDAYAGIRI PILLAR WITH ANIMALS / BIRDS AT VERTICES OF AN OCTAGON

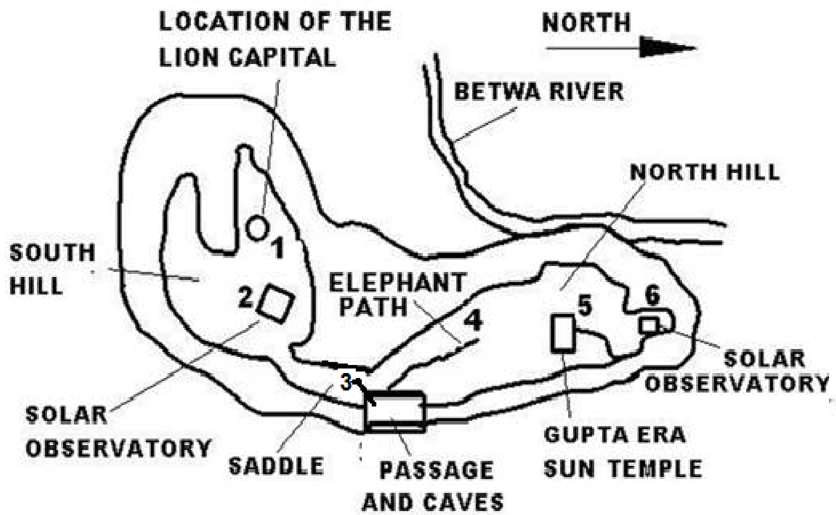

FIG.2A GENERAL PLAN VIEW OF THE ASTRONOMICAL SITE AT UDAYAGIRI

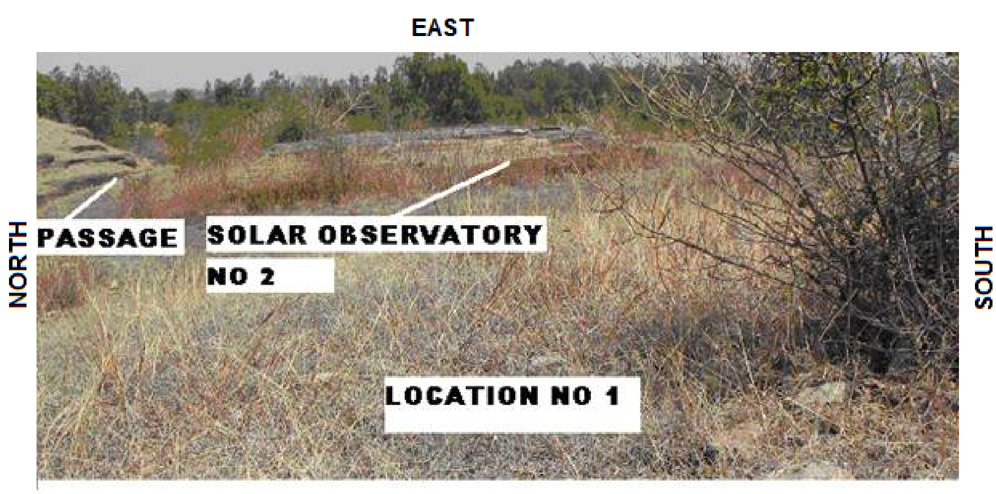

FIG. 2B A VIEW SHOWING RELATIVE LOCATIONS ON THE SOUTH HILL

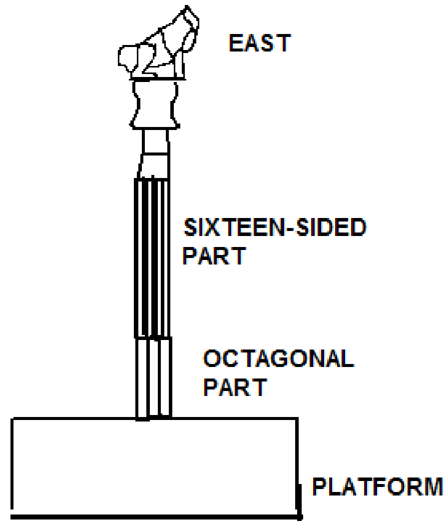

FIG. 3 UDAYAGIRI PILLAR MOUNTED ON A RAISED PLATFORM

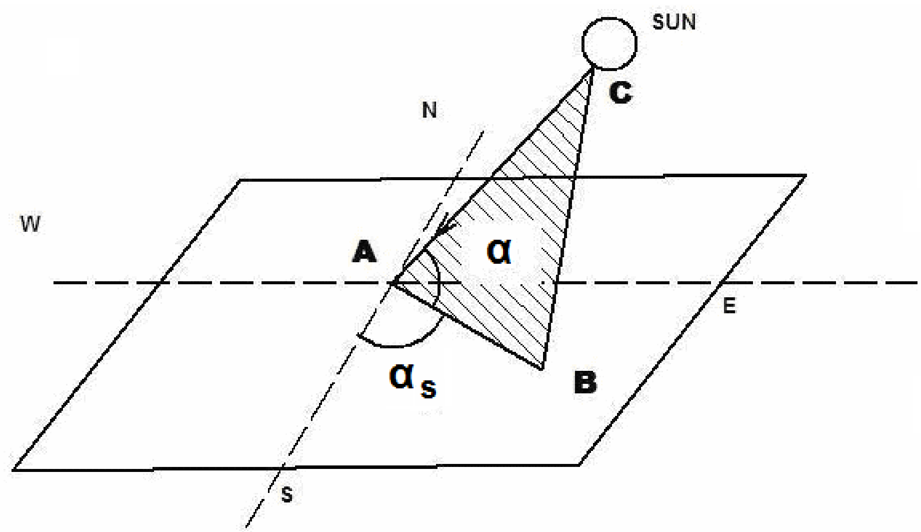

FIG. 4 DIAGRAM SHOWING THE SOLAR RAY DIRECTION USING TWO ANGLES

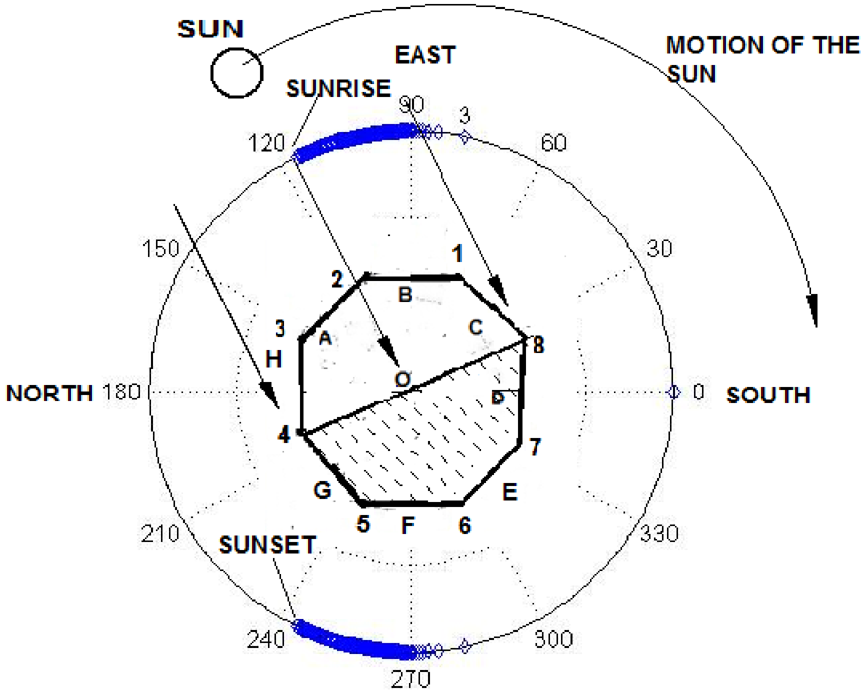

FIG 5 SUN'S RAYS INTERSECTING FOUR SIDES ON THE SUMMER SOLSTICE DAY

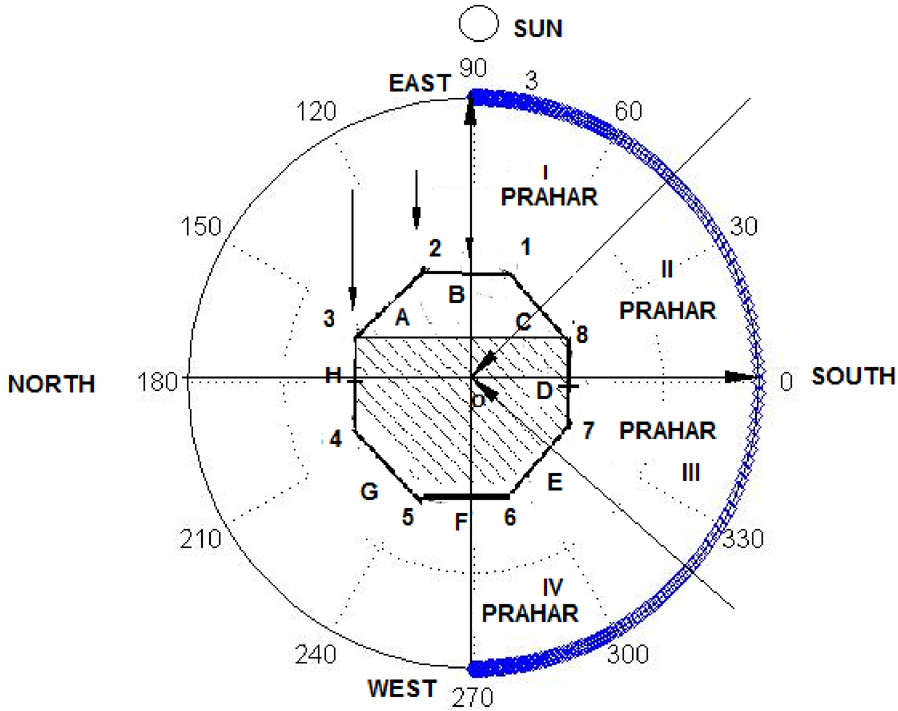

FIG. 6 FOUR EQUAL PRAHARS ON EQUINOX

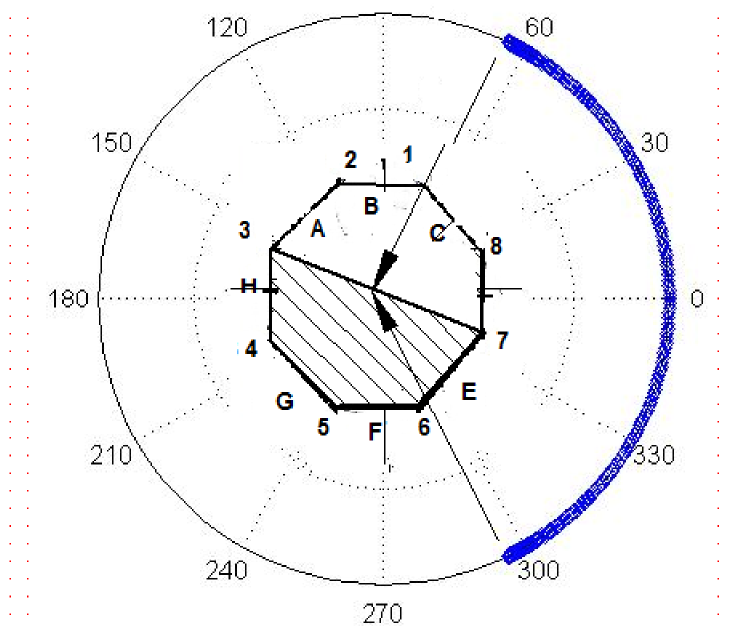

FIG. 7 THE FOUR PRAHARS OF DAY WILL BE SMALLER THAN THE CORRESPONDING ONES OF NIGHT

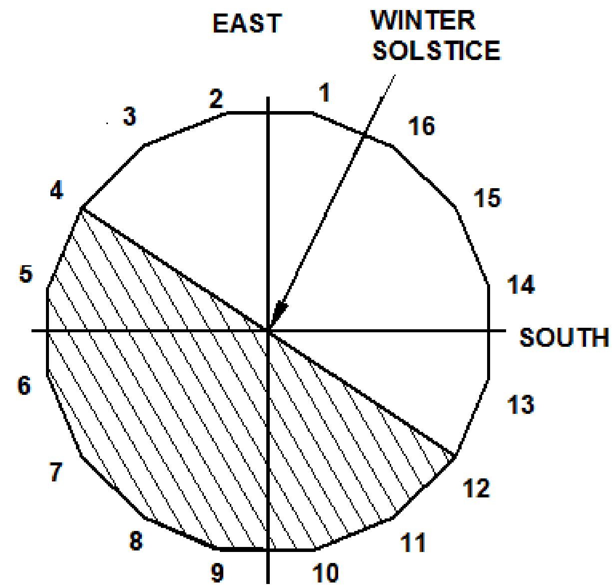

FIG. 8 DARKER SIDES DURING WINTER SOLSTICE USING 16 SIDED POLYGON

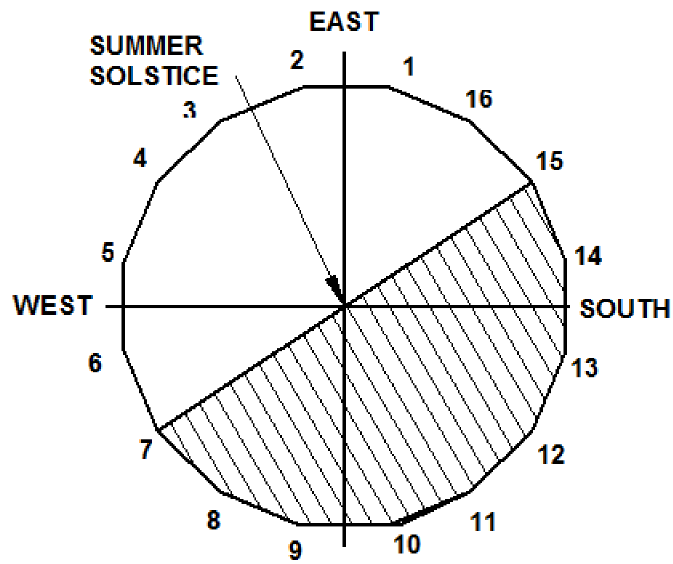

FIG. 9 DARKER SIDES DURING THE SUMMER SOLSTICE USING 16 SIDED POLYGON

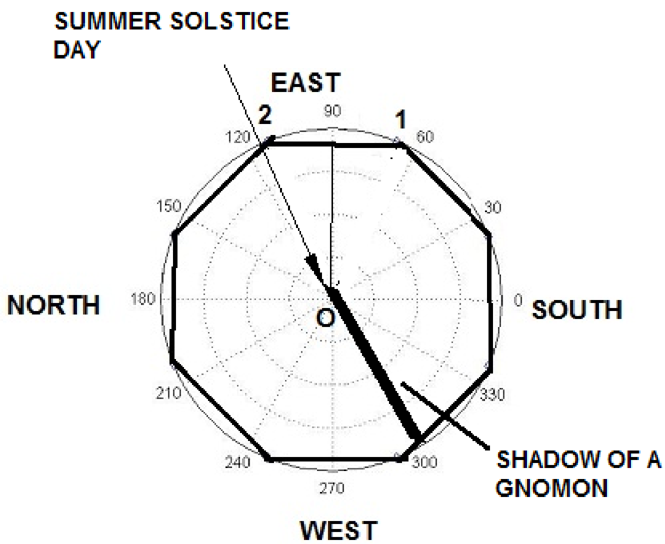

FIG 10 SHADOW OF A GNOMON ON THE SUMMER SOLSTICE DAY

FIG. 11 A SANKU YANTRA IN USE AT UJJAIN

FIG. 12 A CLOSER VIEW OF THE PLATFORM AT LOCATION NUMBER 2 ON THE SOUTH HILL